N-Step Impacted-Region Optimization based Distributed Model Predictive Control ⋆

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Abstract: The Distributed Model Predictive Control (DMPC) has been more and more popular in the control of distributed systems which are composed by many interacted subsystems. The range of subsystems that each local Model Predictive Control (MPC) optimized, called coordination degree, plays an important role in improving the optimization performance of entire closed-loop system. In this paper, the N-step adjacent structure matrix based decomposition method was proposed, where the coordination degree of each subsystem is determined by the union of the all the adjacent matrices over the predictive horizon. Based on this decomposition, each local MPC considers the cost of all the subsystems it impacted on during the predictive horizon, and then improves the optimization performance of entire system with reduced communication burdens. The simulation results show the effectiveness of the proposed method.

1. INTRODUCTION

Consider a class of complex large-scale control systems which is composed of many physically or geographically divided subsystems. Each subsystem interacts with some other subsystems by their states and/or inputs. The control objective is to accomplish a specific global performance of the entire system or a common goal of all subsystems.

The Distributed Model Predictive Control (DMPC) which controls each subsystem by a separated local Model Predictive Control (MPC), has been more and more popular in the control of this kinds of systems [Moroşan et al., 2010], since it not only inherits MPC's advantages of explicitly accommodating constraints and good dynamic performance, but also has all the virtues of distributed framework[Qin and Badgwell, 2003, Maciejowski, 2002, Sandell Jr et al., 1978, Scattolini, 2009, Leirens et al., 2010, Christofides et al., 2012, Zheng et al., 2011b, 2013a].

However, the performance of distributed implementation of MPC, in most of cases, is not as good as that of centralized MPC. To improve the global performance of entire closed-loop system, several DMPC coordination strategies appeared in the literatures [Zheng et al., 2009, Camponogara et al., 2002, Christofides et al., 2012, Zheng et al., 2013b]. People found that if the subsystems of each MPC's cost function covered, called coordination degree, increased, the optimization performance of entire system is improved and the communication burden increases in most cases [Al-Gherwi et al., 2010]. Specially, if each local MPC optimizes its own cost function, uses the predictive sequence of its neighbors to estimate the interactions among subsystems [Camponogara et al., 2002], and employs iterative algorithm, the Nash optimality can be achieved[Li et al., 2005]. In this strategy, each local controller connects to its neighboring controllers. To farther improve the global performance, a design method that each subsystem-based MPC takes not only the performance of its corresponding subsystem but also that of the subsystems it directly impacts on into account in its optimization index is proposed by [Li et al., 2014, Zheng et al., 2009]. Experiments and numeric results prove that this strategy could significantly improve the performance of entire system with small increasing of network connectivity, each local controller have to connect to the controllers of its neighbours, and its neighbours's neighbours. Another useful strategy is that each subsystem optimize the weighted cost of all subsystems and solves the optimal solution by parallel iteration. By this method, the Parato Optimality, the best optimization performance in existing DMPCs, can be achieved[Stewart et al., 2010, Zheng et al., 2011a]. However, the global information is required when solving each subsystem's optimal solution in this method, which is not expected. Could we find a method to exactly define the coordination degree, which is able to make the resulting DMPC obtains Pareto optimality with reduced communication burden? This stimulated this study.

In this paper, an N-step Impacted Region Optimization based DMPC (N-step IRO-DMPC) is proposed, where

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the range of each local MPC's cost function covered is determined by the union of the all the adjacent matrices over the predictive horizon. Each local MPC optimized the cost of all subsystems it impacted on over the prediction horizon to cooperate with each other. In addition, an iterative algorithm is developed to resolve each local MPC. Through these ways, the Parato optimality was achieved with reduced communication resource comparing to the global cost optimization based DMPC.

The remainder of this paper is organized as follows. Section 2 describes the problem to be solved in this paper. Section 3 presents the design of the proposed distributed MPC. Section 4 presents the simulation results to demonstrate the effectiveness of the proposed method. Finally, a brief conclusion to the paper is drawn in Section 5.

2. PROBLEM

2.1 Distributed system

A distributed system is composed of many interacting subsystems, each of which is controlled by an independent controller, which in turn is able to exchange information with other controllers.

Suppose that the distributed system $\mathcal{S}$ is composed of m discrete-time linear subsystems $\mathcal{S}_i$, $i \in \mathcal{P} = \{1, 2, \cdots, m\}$ and m controllers $\mathcal{C}_i$, $i \in \mathcal{P}$. If subsystem $\mathcal{S}_i$ is directly affected by $\mathcal{S}_j$, for any $i \in \mathcal{P}$ and $j \in \mathcal{P}$, subsystem $\mathcal{S}_i$ is said to be a directly (or one-step) downstream system of subsystem $\mathcal{S}_j$, and subsystem $\mathcal{S}_j$ is a directly (or one-step) upstream system of $\mathcal{S}_i$. Let $\mathcal{P}_{+i}$ denote the set of the subscripts of the one-step upstream systems of $\mathcal{S}_i$, $\mathcal{P}_{-i}$ is the set of the subscripts of the one-step downstream systems of $\mathcal{S}_i$. Let the subsystems interact with each other through their states. Then, subsystem $\mathcal{S}_i$ can be expressed as

$$\begin{cases} \mathbf{x}_{i,k+1} = \mathbf{A}_{ii}\mathbf{x}_{i,k} + \mathbf{B}_{ii}\mathbf{u}_{i,k} + \sum\limits_{j\in\mathcal{P}_{+i}} \mathbf{A}_{ij}\mathbf{x}_{j,k}, \\ \mathbf{y}_{i,k} = \mathbf{C}_{ii}\mathbf{x}_{i,k}, \end{cases} \quad (1)$$

where $\mathbf{x}_i \in \mathcal{X}_i \subset \mathbf{R}^{n_{xi}}$, $\mathbf{u}_i \in \mathcal{U}_i \subset \mathbf{R}^{n_{ui}}$ and $\mathbf{y}_i \in \mathcal{Y}_i \subset \mathbf{R}^{n_{yi}}$ are respectively the local state, input and output vectors, and $\mathcal{X}_i$, $\mathcal{U}_i$ and $\mathcal{Y}_i$ are respectively the feasible set of the state $\mathbf{x}_i$, input $\mathbf{u}_i$ and output $\mathbf{y}_i$ which are used to bound the state, input and output according to the physical constraints on the actuators, the control requirements or the characteristics of the plant. A non-zero matrix $\mathbf{A}_{ij}$, $j \in \mathcal{P}_{+i}$, indicates that $\mathcal{S}_i$ is directly affected by $\mathcal{S}_j$. In the concatenated vector form, the system dynamics can be written as

$$\begin{cases} \mathbf{x}_{a,k+1} = \mathbf{A}\mathbf{x}_{a,k} + \mathbf{B}\mathbf{u}_{a,k}, \\ \mathbf{y}_{a,k} = \mathbf{C}\mathbf{x}_{a,k}, \end{cases} \quad (2)$$

where $\mathbf{x}_a = [\mathbf{x}_1^{\mathrm{T}}\ \mathbf{x}_2^{\mathrm{T}}\ \cdots\ \mathbf{x}_m^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{R}^{n_x}$, $\mathbf{u}_a = [\mathbf{u}_1^{\mathrm{T}}\ \mathbf{u}_2^{\mathrm{T}}\ \cdots\ \mathbf{u}_m^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{R}^{n_u}$ and $\mathbf{y}_a = [\mathbf{y}_1^{\mathrm{T}}\ \mathbf{y}_2^{\mathrm{T}}\ \ldots\ \mathbf{y}_m^{\mathrm{T}}]^{\mathrm{T}} \in \mathbf{R}^{n_y}$ are respectively the concatenated state, control input and output vectors of the overall system $\mathcal{S}$, and $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are constant matrices of appropriate dimensions. Also, $\mathbf{x}_a \in \mathcal{X} = \mathcal{X}_1 \times \mathcal{X}_2 \times \cdots \times \mathcal{X}_m$, $\mathbf{u}_a \in \mathcal{U} = \mathcal{U}_1 \times \mathcal{U}_2 \times \cdots \times \mathcal{U}_m$ and $\mathbf{y}_a \in \mathcal{Y} = \mathcal{Y}_1 \times \mathcal{Y}_2 \times \cdots \times \mathcal{Y}_m$.

2.2 Control objective

The control objective is to find a control method under the distributed framework, which could obtain the optimal solution of entire system with a reduced communication burden. The performance index of the whole system can be expressed as

$$J(k) = \sum_{i\in\mathcal{P}} J_i(k) \quad (3)$$

where

$$J_i(k) = \sum_{l=1}^{N} \Big(\big\|\mathbf{C}_{ii}\mathbf{x}_{i,k+l|k} - \mathbf{y}_i^{sp}\big\|^2_{\mathbf{Q}_{i,l}} + \big\|\Delta\mathbf{u}_{i,k+l-1|k}\big\|^2_{\mathbf{R}_{i,l}} \Big), (4)$$

$\mathbf{y}_i^{sp}$ is the set point of the i^{th} subsystem and $\Delta\mathbf{u}_{i,k+l|k} = \mathbf{u}_{i,k+l|k} - \mathbf{u}_{i,k+l-1|k}$ is the input increment of the i^{th} subsystem at the time instant k. Constant matrix $\mathbf{Q}_{i,l}, \mathbf{R}_{i,l} > 0$, $l = 1, 2, \ldots, N$, is weighting coefficients for the i^{th} subsystem, and let the weighting matrices for $\mathcal{S}_i$ be

$$\mathbf{Q}_i = \mathrm{block} - \mathrm{diag}\{\mathbf{Q}_{i,1}, \mathbf{Q}_{i,2}, \ldots, \mathbf{Q}_{i,N}\} > 0$$
$$\mathbf{R}_i = \mathrm{block} - \mathrm{diag}\{\mathbf{R}_{i,1}, \mathbf{R}_{i,2}, \ldots, \mathbf{R}_{i,N}\} > 0.$$

3. N-STEP IMPACTED-REGION OPTIMIZATION BASED DISTRIBUTED MPC

Consider that each local MPC $\mathcal{C}_i, i \in \mathcal{P}$, only optimize N (predictive horizon) step ahead performance, thus the solution of each local MPC only impacts performance of the subsystems which are interacted with $\mathcal{S}_i$ during the predictive horizon. To take this interaction into account, a strategy that each local MPC optimize its N-step impacted region's performance was proposed, and which is detailed as follows.

3.1 N-step impacted-region

To proceed, we need the following definitions.

Definition 1. Adjacent Matrix: Consider a system $\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k$, $\mathbf{A} \in \mathcal{R}^{n_x \times n_x}$ which is composed of m subsystem, the adjacent matrix refers to a $m \times m$ matrix $\bar{\mathbf{A}}$, and its i^{th}, j^{th} element

$$\bar{a}_{ij} = \begin{cases} 1, & \text{when } \mathbf{A}_{ij} \neq \mathbf{0} \\ 0, & \text{when } \mathbf{A}_{ij} = \mathbf{0} \end{cases} \quad (5)$$

It can be seen from (5) that the adjacent matrix only reflects the directly interaction among subsystems. In fact, we usually concern that if one unit indirectly impacts another unit through some intermediate units. In this case, the structure matrix corresponding to the k power of $\bar{\mathbf{A}}$ can be used to express the interaction in k step ahead. Then, following N-Step Accessible Matrix is defined to express the relationship among subsystem during N step ahead.

Definition 2. N-Step Accessible Matrix: Consider a system $\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k$, $\mathbf{A} \in \mathcal{R}^{n_x \times n_x}$, which consists of m unit $\mathcal{S}_i$, $i = 1, 2, ..., m$, define that each unit $\mathcal{S}_i$ is accessible to itself, the all accessible relationship can be described by a so called N-Step Accessible Matrix $\mathbf{R}$, and

$$\mathbf{R} = \mathbf{I} \cup \bar{\mathbf{A}} \cup \bar{\mathbf{A}}^2 \cup \ldots \cup \bar{\mathbf{A}}^N$$

It is also a kind of structure matrix, where the i^{th} row and j^{th} column element equals zero expresses that subsystem

$\mathcal{S}_j$ is un-accessible to subsystem $\mathcal{S}_j$. By logic operating rules, the above equation can be simply rewritten as

$$\mathbf{R} = (\mathbf{I} \cup \bar{\mathbf{A}})^N \tag{6}$$

In fact, for the control of system (1), the relationship among the states, inputs and outputs are very important. However, (5) can not exactly reflect this relationship. Consider that

$$\begin{bmatrix} \mathbf{x}_{a,k+1} \\ \mathbf{u}_{a,k+1} \\ \mathbf{y}_{a,k} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{C} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{a,k} \\ \mathbf{u}_{a,k} \\ \mathbf{y}_{a,k-1} \end{bmatrix}, \tag{7}$$

for system (2), the adjacent matrix can be defined as

$$\bar{\mathbf{A}}_d = \begin{bmatrix} \bar{\mathbf{A}} & \bar{\mathbf{B}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \bar{\mathbf{C}} & \mathbf{0} & \mathbf{0} \end{bmatrix}. \tag{8}$$

From (6), the N-step Accessible Matrix can be expressed as

$$\begin{aligned} \mathbf{R} = (\mathbf{I} \cup \mathbf{A}_d)^{N-1} &= \begin{bmatrix} \mathbf{R}_{xx} & \mathbf{R}_{xu} & \mathbf{R}_{xy} \\ \mathbf{R}_{ux} & \mathbf{R}_{uu} & \mathbf{R}_{uy} \\ \mathbf{R}_{yx} & \mathbf{R}_{yu} & \mathbf{R}_{yy} \end{bmatrix} \\ = \begin{bmatrix} (\bar{\mathbf{A}}+\mathbf{I})^N & \bar{\mathbf{B}}(\bar{\mathbf{A}}+\mathbf{I})^{N-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \bar{\mathbf{C}}(\bar{\mathbf{A}}+\mathbf{I})^{N-1} & \bar{\mathbf{C}}(\bar{\mathbf{A}}+\mathbf{I})^{N-2}\bar{\mathbf{B}} & \mathbf{I} \end{bmatrix}. \end{aligned} \tag{9}$$

Thus, the N-step input to output accessible matrix is

$$\mathbf{R}_{yu} = \bar{\mathbf{C}}(\bar{\mathbf{A}}+\mathbf{I})^{N-2}\bar{\mathbf{B}}. \tag{10}$$

Consider that $\bar{\mathbf{B}}$ is a unit matrix in (10), the N-step accessible matrix can be redefined as

$$\mathbf{R}_{yu} = \bar{\mathbf{C}}(\bar{\mathbf{A}}+\mathbf{I})^{N-2}. \tag{11}$$

And the N-step down stream neighbour of $\mathcal{S}_i$ can be defined as the subsystems $\mathcal{S}_j$ where the j^{th} row, and i^{th} collum element of $\mathbf{R}_{yu}$ equals 1. And denote $\mathcal{P}_{-iN}$ as the set of the subscript of all the N-step downstream neighbor of $\mathcal{S}_i$.

3.2 Distributed MPC design

Consider that the control law of current subsystem $\mathcal{S}_i$ effects the performance of its N-step downstream neighboring subsystems $\mathcal{S}_j, j \in \mathcal{P}_{-iN}$, in the N-Step IRO-DMPC, the performance of $\mathcal{S}_j$, $j \in \mathcal{P}_{-iN}$ is added into the performance index of the MPC which controls $\mathcal{S}_i$ based on a approximation of the updated state sequence of $\mathcal{S}_j$. In this way, the coordination degree is expanded and is equivalent to that of global cost optimization based DMPC.

The performance of local MPC for subsystem $\mathcal{S}_i$ is defined as

$$\begin{aligned} \bar{J}_i(k) = &\sum_{l=1}^{N} \left(\left\| \mathbf{C}_{ii}\mathbf{x}_{i,k+l|k} - \mathbf{y}_i^{sp} \right\|^2_{\mathbf{Q}_{i,l}} + \left\| \Delta\mathbf{u}_{i,k+l-1|k} \right\|^2_{\mathbf{R}_{i,l}} \right) \\ &+ \sum_{j\in\mathcal{P}_{-iN}} \sum_{l=1}^{N} \left\| \mathbf{C}_{jj}\hat{\mathbf{x}}_{j,k+l|k} - \mathbf{y}_j^{sp} \right\|^2_{\mathbf{Q}_{j,l}} \end{aligned} \tag{12}$$

Define that $\hat{\mathbf{x}}_{i,k+l|k}$, $\hat{u}_{i,k+l|k}$ and $\Delta\hat{u}_{i,k+l|k}$ be the assumed states, the assumed input and the assumed input increment which are calculated in the previous calculation, respectively.

The predictive model can be expressed as

$$\begin{aligned} \mathbf{y}_{i,k+l|k} = &\mathbf{C}_{ii}\mathbf{A}_{ii}^{l}\mathbf{x}_{i,k} + \sum_{h=1}^{l} \mathbf{C}_{ii}\mathbf{A}_{ii}^{l-h}\mathbf{B}_{ii}u_{i,k+h-1|k} \\ &+ \sum_{j\in\mathcal{P}_{+i}} \sum_{h=1}^{l} \mathbf{C}_{ii}\mathbf{A}_{ii}^{l-h}\mathbf{A}_{ij}\hat{\mathbf{x}}_{j,k+h-1|k} \end{aligned} \tag{13}$$

Consider the physical limitations on the outputs, the input and the input increment, we can get following optimization problem for $\mathcal{S}_i$ in each control period.

Problem 1. For all subsystem $\mathcal{S}_i$, $i \in \{1,2,\ldots,m\}$, provided that $\mathbf{x}_{i,k}$, $\hat{\mathbf{x}}_{j,k+l|k}$, $j \in \mathcal{P}_{+h} \bigcup P_{+i}, h \in \mathcal{P}_{-iN}$ and $\Delta\hat{\mathbf{u}}_{i,k+l-1|k-1}, j \in \mathcal{P}_{-iN}$, $l = 1,2,\ldots,N$,
find the control sequence $\Delta\mathbf{u}_{i,k:k+N-1|k}$, which minimize the performance index

$$\min_{\mathbf{u}_{i,k:k+N-1|k}} \bar{J}_i(k)$$

Subject to the constraints:

$$\begin{aligned} &\text{Equation(13)}, \\ &\mathbf{y}_{i,L} \le \mathbf{y}_{i,k+l|k} \le \mathbf{y}_{i,U}, && (14) \\ &\mathbf{y}_{j,L} \le \mathbf{y}_{j,k+l|k} \le \mathbf{y}_{j,U}, j \in \mathcal{P}_{-iN}, && (15) \\ &\mathbf{u}_{i,L} \le \mathbf{u}_{i,k+l-1|k} \le \mathbf{u}_{i,U}, && (16) \\ &\Delta\mathbf{u}_{i,L} \le \Delta\mathbf{u}_{i,k+l-1|k} \le \Delta\mathbf{u}_{i,U}, && (17) \\ &l = 1,2,...,N; \\ &||\mathbf{y}_{i,k+N|k} - \mathbf{y}_i^{sp}||^2_{\mathbf{Q}_{i,N}} < \varepsilon^2. && (18) \end{aligned}$$

where, $[\mathbf{y}_{i,L}, \mathbf{y}_{i,U}]$, $[\mathbf{u}_{i,L}, \mathbf{u}_{i,U}]$ and $[\Delta\mathbf{u}_{i,L}, \Delta\mathbf{u}_{i,U}]$ are the bounds of outputs, inputs and the increment of inputs respectively. Equation (18) is a final constraint for improve the stability of each subsystem-based MPC, and $\varepsilon > 0$.

To solve problem (1) efficiently, following iterative algorithm is given for $\forall\mathcal{S}_i$, $i \in \mathcal{P}$.

Algorithm 1. (N-step IRO-DMPC Algorithm).

Step 1: Initialization.

- Initialize $\mathbf{x}_{i,k_0}$, $\mathbf{x}_{i,k_0+l|k_0}$, $l = 1,2,\ldots,N$, which satisfy the constraints of Problem 1.

Step 2: Update control law at time $k > k_0$.

- Step 2.1
  Set iteration $t = 1$, and set $\hat{\mathbf{x}}_{i,k+l|k} = \mathbf{x}_{i,k+l|k-1}$.
- Step 2.2
  Measure $x_i(k)$, transmit $\hat{\mathbf{x}}_{i,k+l|k}$ to its N-step down stream neighbors and $\hat{\mathbf{u}}_{i,k+l|k}$ upstream neighbors; And receive $\hat{\mathbf{u}}_{i,k+l|k}$ from its N-step down stream neighbors and $\hat{\mathbf{x}}_{j,k+l|k}$ from its upstream neighbors.
- Step 2.3
  Solving Problem 1 to obtain the optimal solution $\Delta\mathbf{u}^t_{i,k+l|k}$, and predict the future state $\mathbf{x}_{i,k+l|k}$ based on the solution $\Delta\mathbf{u}^t_{i,k+l|k}$.
- Step 2.4
  If
  $$||\Delta\mathbf{u}^t_{i,k+l-1|k} - \Delta\mathbf{u}^{t-1}_{i,k+l-1|k}||^2_2 \le \varepsilon_0 \ \text{ or } \ t > t_{max}$$
  then set
  $$\mathbf{u}^*_{i,k} = \mathbf{u}_{i,k-1} + \Delta\mathbf{u}^*_{i,k+l-1|k},$$

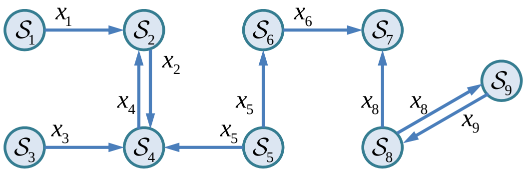

Fig. 1. Distributed MPC configuration

and goto Step 3; Else set

$$\hat{\mathbf{x}}_{i,k+l|k} = \mathbf{x}_{i,k+l|k}, t = t + 1,$$

and goto Step 2.2.

Step 3: Update control at time $k+1$.

- Let $k+1 \to k$, repeat Step 2.

It should be noticed that although an iterative algorithm is presented, the Problem 1 can also be solved by a non-iterative algorithm through setting $t_{max} = 1$. Since the communication burden will increase with the increasing of iteration, t_{max} should not be set too large in practice. So far the N-step impacted-region optimization based DMPC for distributed system is introduced, some simulation results will be presented in the next section to show the effectiveness of the proposed method.

4. SIMULATION

For simplicity, a 9 nodes distributed network is taken as example, and the relationship among these notes is shown in Fig. 1 where the arrow from subsystem $\mathcal{S}_i$ to $\mathcal{S}_j$ expresses that $\mathcal{S}_j$ is directly effected by $\mathcal{S}_i$.

The dynamic models of these nodes are respectively given by

$$\mathcal{S}_1 : \begin{cases} x_{1,k+1} = 0.57x_{1,k} + 0.38u_{1,k} \\ y_{1,k} = x_{1,k}, \end{cases} \quad (19)$$

$$\mathcal{S}_2 : \begin{cases} x_{2,k+1} = 0.53x_{2,k} + 0.38u_{2,k} + 0.16x_{1,k} \\ \qquad\qquad +0.16x_{4,k} \\ y_{2,k} = x_{2,k}, \end{cases} \quad (20)$$

$$\mathcal{S}_3 : \begin{cases} x_{3,k+1} = 0.55x_{3,k} + 0.38u_{3,k} \\ y_{3,k} = x_{3,k}, \end{cases} \quad (21)$$

$$\mathcal{S}_4 : \begin{cases} x_{4,k+1} = 0.61x_{4,k} + 0.39u_{4,k} + 0.18x_{2,k} \\ \qquad\qquad +0.18x_{3,k} + 0.18x_{5,k} \\ y_{4,k} = x_{4,k}, \end{cases} \quad (22)$$

$$\mathcal{S}_5 : \begin{cases} x_{5,k+1} = 0.68x_{5,k} + 0.42u_{5,k} \\ y_{5,k} = x_{5,k}, \end{cases} \quad (23)$$

$$\mathcal{S}_6 : \begin{cases} x_{6,k+1} = 0.55x_{6,k} + 0.38u_{6,k} + 0.16x_{5,k} \\ y_{6,k} = x_{6,k}, \end{cases} \quad (24)$$

$$\mathcal{S}_7 : \begin{cases} x_{7,k+1} = 0.71x_{7,k} + 0.42u_{7,k} + 0.21x_{6,k} \\ \qquad\qquad +0.21x_{8,k} \\ y_{7,k} = x_{7,k}, \end{cases} \quad (25)$$

$$\mathcal{S}_8 : \begin{cases} x_{8,k+1} = 0.57x_{8,k} + 0.38u_{8,k} + 0.17x_{9,k} \\ y_{8,k} = x_{8,k}, \end{cases} \quad (26)$$

$$\mathcal{S}_9 : \begin{cases} x_{9,k+1} = 0.66x_{9,k} + 0.41u_{9,k} + 0.20x_{8,k} \\ y_{9,k} = x_{9,k}. \end{cases} \quad (27)$$

According to (11), the N-step accessible matrix is:

$$\mathbf{R}_{yu} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0 \\ 1&1&1&1&1&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0 \\ 1&1&1&1&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&1&1&0&0&0 \\ 0&0&0&0&1&1&1&1&1 \\ 0&0&0&0&0&0&0&1&1 \\ 0&0&0&0&0&0&0&1&1 \end{bmatrix} \quad (28)$$

From (28) the network connectivity is 15 when using proposed N-step ICO-DMPC, which is most less than 72 when using global cost optimization based DMPC. The network connectivity is dramatically deduced.

For the purpose of comparison, both the Centralized MPC and the N-step IRO-DMPC are applied to this system. Let the constraint on the input be $[u_{i,\mathrm{L}}, u_{i,\mathrm{U}}] = [-2, 2]$ and the constraint on the increment of input be $[\Delta u_{i,\mathrm{L}}, \Delta u_{i,\mathrm{U}}] = [-1.5, 1.5]$. Set the all controllers' (both centrialized MPC and N-step IRO-DMPC) parameters of control horizon be $N = 10$, the weighting matrices be $\mathbf{Q}_i = [1, 1, 1, 1, 1, 1, 1, 1, 1, 5]$, $\mathbf{R}_i = [1, 1, 1, 1, 1, 1, 1, 1, 1, 1]$, where $i \in \{1, 2, ..., 9\}$.

The state responses and the inputs of the closed-loop system under the control of the centralized MPC and LCO-DMPC are shown in Figs. 2 and 3, respectively. The shape of the state response curves under the control of N-step IRO-DMPC are almost equals to those under the Centralized MPC. Under the N-step IRO-DMPC control design, when set point changed, there is no significant over-shooting, but some fluctuation exists in the trajectories of states of the interacting subsystems.

From these simulation results, it can be seen that the proposed N-step IRO-DMPC could obtain a global performance almost equal to that of using centralized MPC and the global information is not necessary for every local MPC, which keeps the characteristics of good error tolerance and high flexibility of the Distributed Control Framework.

5. CONCLUSION

In this paper, an N-step Impact-Region Optimization based DMPC is provided for distributed systems. The simulation results of the control of a distributed network composed by nine first-order systems shows the efficiency of the proposed method. With the proposed method, the closed-loop system could obtain a global performance almost equivalent to that of with the centralized MPC. In addtion, the global information is not necessary for each local MPC in the N-Step IRO-DMPC comparing to the global cost optimization based DMPC, which could significantly deduce the network-connectivity for sparse systems, increase the capability of error tolerance of control system. The stabilizing implementation of proposed DMPC subject to pdecoupled constraints maybe a extension of this work and will be done in the near future.

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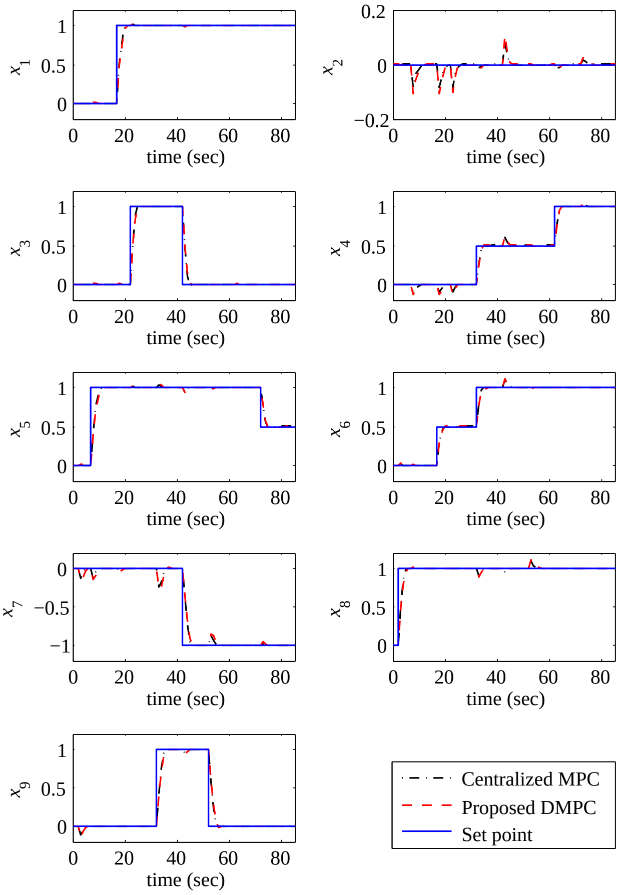

Fig. 2. The evolution of the states under the centralized MPC, and N-step Impated-region optimization based DMPC.

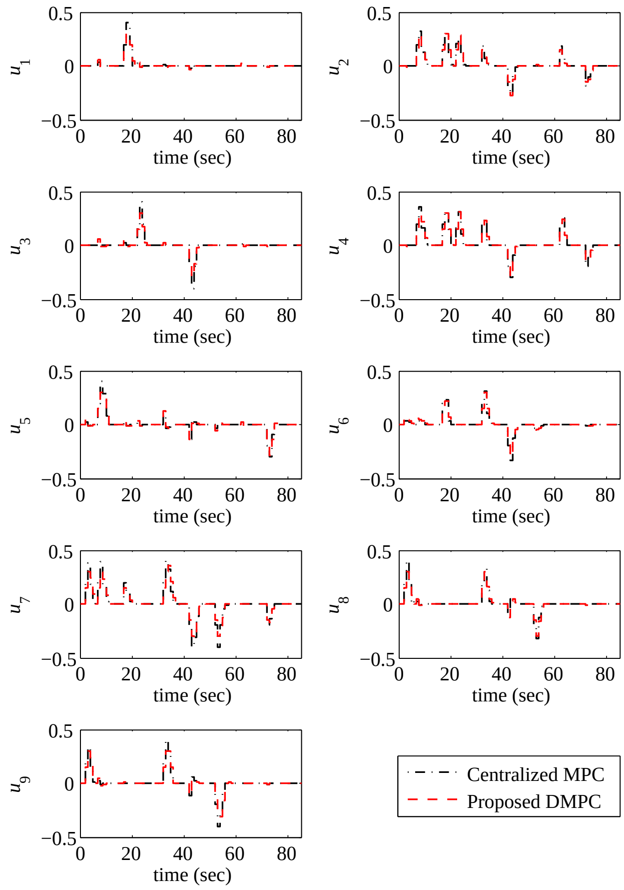

Fig. 3. The evolution of the inputs under the centralized MPC, and N-step Impated-region optimization based DMPC.

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